Downloaded from rsta.royalsocietypublishing.org

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A
MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Instabilities in the Solar Wind

S. J. Schwartz and I. W. Roxburgh

Phil. Trans. R. Soc. Lond. A 1980 **297**, 555-563
doi: 10.1098/rsta.1980.0231

Email alerting service Receive free email alerts when new articles cite this article - sign up in the box at the top right-hand corner of the article or click **here**

To subscribe to *Phil. Trans. R. Soc. Lond. A* go to: **http://rsta.royalsocietypublishing.org/subscriptions**

This journal is © 1980 The Royal Society

Downloaded from rsta.royalsocietypublishing.org

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Phil. Trans. R. Soc. Lond. A **297**, 555–563 (1980) [555]
Printed in Great Britain

Instabilities in the solar wind

By S. J. Schwartz and I. W. Roxburgh
Department of Applied Mathematics, Queen Mary College, University of London, Mile End Road, London E1 4NS, U.K.

We review recent progress in the possible role of microturbulence in the solar wind. The solar wind is expected to excite plasma microinstabilities owing to its transition from a collision-dominated to a collisionless plasma, with potentially drastic consequences for thermal transport and other physical processes. We discuss both the extensive linear theory of this subject and also our present understanding of nonlinear plasma turbulence. The solar wind is an excellent laboratory for studying many aspects of solar and plasma physics, and may soon provide some answers to several fundamental questions.

1. Introduction

Throughout most of the interplanetary medium (beyond, say, 10–20 solar radii from the Sun) collisions between solar wind (s.w.) particles are sufficiently rare that many physical processes can only be described in the context of a collisionless plasma rather than a fluid. In other words, the microscopic details of the particle distribution may result in considerably different physics (e.g. thermal conduction laws) to that derived under 'classical' collision-dominated fluid approximations. Thus, even if the s.w. were a time-stationary radial expansion of a spherically symmetric electron–proton plasma, its microstructure would still be of considerable interest. This review is devoted to the study of microinstabilities which might be expected to develop in such an expansion due to electron thermal conduction and/or temperature anisotropies. The present work is a condensed version of Schwartz (1979, 1980). Other reviews, which cover various aspects of waves and instabilities in the s.w., include Barnes (1979), Burlaga (1971), Feldman (1979), Hollweg (1974*c*, 1975, 1978*b*), Hundhausen (1972), Jokipii (1973), Lee & Lerche (1974), Scarf (1970) and Völk (1975).

Basically, classical collision-dominated heat conduction (Spitzer & Härm 1953) predicts the small deviation of a plasma from a Maxwellian distribution in the presence of a temperature gradient. As the s.w. expands, however, collisions become too rare to keep this skewness small. This heat current may then drive plasma instabilities which in turn provide alternative sources of particle scattering, thereby regulating the heat flux and governing the particle distribution. In addition, the conservation of collisionless adiabatic invariants (Chew *et al.* 1956) implies that the kinetic particle temperature perpendicular to the magnetic field should fall considerably below the parallel temperature as the s.w. expands (Hollweg 1970, 1971*a*; Jockers 1970). This firehose-type distribution may also lead to microinstabilities.

In §2 we compile and discuss the various instabilities that have been considered in s.w. literature. Section 3 briefly introduces various nonlinear plasma theories which are or may be needed to incorporate these instabilities in a quantitative way into solar wind physics. Section 4 provides some speculative consequences of microinstabilities before the conclusions of §5.

The Royal Society is collaborating with JSTOR to digitize, preserve, and extend access to *Philosophical Transactions of the Royal Society of London. Series A, Mathematical and Physical Sciences.* www.jstor.org

Downloaded from rsta.royalsocietypublishing.org

Table 1. Properties of solar wind microinstabilities

instability	dispersion relation, ω	propagation direction	model	driven by	notes
whistler (W), $\omega \approx \Omega_i + (\Omega_e/\omega_{pe}^2)\, c^2k^2 \cos\theta$					
ASF*a*		antisolar $\Vert \boldsymbol{B}_0$	bi-Lorentzian	drift	γ sensitive to halo shape: threshold sensitive to halo anisot.
DT		antisolar $\Vert \boldsymbol{B}_0$	bi-Max. e^- core, halo, ions	halo drift	analytic deriv. displaying many features, $kR_{Le} \sim 0.5$
GEA*a*, GFI		antisolar $\Vert \boldsymbol{B}_0$	Max. e^- core, halo Max. ions	halo-ion drift	quenched by $T_{h\Vert}/T_{h\perp}$ large lower threshold for higher β_i
GEA*b*		antisolar $\Vert \boldsymbol{B}_0$	e^- core, halo	resonant halo e^-	quenched by $T_{h\Vert}/T_{h\perp} > 1$
GEA*c*	$\omega \sim \Omega_i$	$\Vert \boldsymbol{B}_0$	bi-Max. ions	$T_{i\Vert} > T_{i\perp}$	
HV	$\omega \gtrsim \Omega_i$	$\Vert \boldsymbol{B}_0$	bi-Max. e^- & i	$T_{i\Vert} > 4T_{i\perp}$	protons resonant
KS	$\omega \gtrsim \Omega_i$	$\Vert \boldsymbol{B}_0$	general	$T_{i\Vert} > T_{i\perp}$	long wavelength limit 'RH cyclotron inst.'
MEA*a*, *b*	$\omega \sim \Omega_i$	$\Vert \boldsymbol{B}_0$	beam i, i, e	Ion beam	
SEA	$\omega \gtrsim \Omega_i$	$\Vert \boldsymbol{B}_0$	$\sim$ cold plasma	$T_{i\Vert} > T_{i\perp}$	if mag. moment conserved, unstable $\sim$ 0.8 AU
fast magnetosonic (FMS), $\omega = kv_A$					
F	$\omega \lesssim \Omega_i$	antisolar	Spitzer–Härm	e^- transit time	
GEA*a*	$\omega \sim \Omega_i$	antisolar	e^- core, halo, i drifting	halo-ion drift	quenched for $T_{h\Vert}/T_{h\perp}$ large
GEA*b*	$\omega \lesssim \Omega_i$	antisol., wide range of θ	e^- core, halo, i	e^- at small v_z	no cyclotron res., θ decr. as k incr.; inhibited by large $T_{c\Vert}/T_{c\perp}$
MEA*a*, *b*	$\omega < \Omega_i$	oblique	i beam, i, e,	ion beam	moderate β_i, small range of β_i
SE	$\omega < \Omega_i$	$\sim 50°$	drift. Max.'s		'favoured' at 1 AU
SI	$\omega \sim 0.1\ \Omega_i$	antisolar $\sim 35°$	Spitzer–Härm	e^- transit time	analytical finite β_i and ω corrections
SII	$\omega \lesssim \Omega_i$	antisolar $\Vert \boldsymbol{B}_0$	Spitzer–Härm		marginal stab. calc., $\theta = 0°$ but $\gamma = 0$
SHI			drift e, i	stable	
ion acoustic (IA)					
F	$\omega \lesssim \omega_{pi}$		Spitzer–Härm	drifting e^-	unstable for large T_e/T_i
G	$\omega \lesssim \omega_{pi}$	$\Vert \boldsymbol{B}_0$ (i.e. drift)	drifting Max. e^-	e^- heat flux	more important than e^- beam and ESIC at high T_e/T_i
GFII	$\omega \sim 5\Omega_i$	$\sim 75°$	drifting Max. e^-	electric current	only unstable if elec. current $\neq 0$
electrostatic ion cyclotron (ESIC), $\omega \gtrsim \Omega_i$					
DR	$\omega \sim 1.2\Omega_i$		drifting Max.'s	current	
F	$\omega \gtrsim \Omega_i$		Spitzer–Härm	shift of peak of reduced distrib.	important at low β_i
FKK	$\omega \sim 1.2\Omega_i$	$\sim 80°$	2 isotr. drift Max.'s + heavy i		unstable for $T_e/T_i < 8.5$ while i-acoust. stable; heavy ion cyclo. wave unstable for small current
G	$\omega \sim \Omega_i$	highly oblique	Max. ions, drifting Max. e^-	e^- heat flux	more important than e beam or IA at intermediate T_e/T_i
SE	$\omega \sim 1.5\Omega_i$	$\sim 80°$	stream. Max.'s	$T_{i\perp}/T_{e\Vert}$ small	low β_i
SII	$\omega \sim 1.5\Omega_i$	Sunward $\sim 70°$	Spitzer–Härm		low β_i
SHI	$\omega \gtrsim \Omega_i$	$\sim 80°$	isot. drift e, i Max.'s	current	$\beta_i \sim 0.1$, more angles for low β_i
electromagnetic ion cyclotron (EMIC), $\omega \lesssim \Omega_i$					
ASF*b*	$\omega \lesssim \Omega_i$	$\Vert \boldsymbol{B}_0$	bi-Lorentz. cold e, i, hot i	anisot. beam i	$\mathbf{k} \Vert \boldsymbol{B}_0$ chosen arbitr., γ sensitive to shape
CS	$\omega \lesssim \Omega_i$	$\Vert \boldsymbol{B}_0$	bi-Max.	$T_{i\perp} > T_{i\Vert}$	multi-component plasma study
F	$\omega \lesssim \Omega_i$		Spitzer–Härm		'should be unstable'
FKK	$\omega \sim 0.75\Omega_i$	$\sim 80°$	2 isot. drift. Max.'s		more important than ESIC for $\beta_i > 10^{-3}$

Downloaded from rsta.royalsocietypublishing.org

instability	dispersion relation, ω	propagation direction	model	driven by	notes
electromagnetic ion cyclotron (EMIC), $\omega \lesssim \Omega_i$ (*cont.*)					
GEA*c*	$\omega < \Omega_i$	$\parallel \boldsymbol{B}_0$	bi-Max. i core, halo	$T_{i\perp} > T_{i\parallel}$	$\beta_i = 1$, T_e/T_i not large, additional ion components can enhance instability
LB	$\omega < \Omega_i$	$\parallel \boldsymbol{B}_0$	counterstr. ions, bi-Max. e^-	$T_{i\perp} > T_{i\parallel}$, strming i	some double-ion solar wind streams are unstable
KS	$\omega < \Omega_i$	$\parallel \boldsymbol{B}_0$	general	$T_{i\perp} > T_{i\parallel}$	low β_i
SE	$\omega \lesssim \Omega_i$		counter-stream bi-Max.'s	stable	high β_i
SII	$\omega \lesssim \Omega_i$	sunward $\sim 30°$	Spitzer–Härm		more unstable than FMS at low β_i
SHI	$\omega \lesssim \Omega_i$	$\sim 45°$	isot. drift e, i Max.'s	current	
Alfvén (A), $\omega = k_z v_A$					
GEA*a*, *b*	$\omega \sim 0.5\Omega_i$	sunward $\sim 80°$	e^- core, halo, i Max.'s	core-ion drift	threshold incr. as β_i incr.
HS, SHI		$\sim 70°$	drift e^- current	current	sensitive to T_e/T_i, wider range of θ for larger β
MEA*a*, *b*		$\sim 30°$	beam i, i, e	ion beam	low β_i
Slow Magnetosonic (SMS), $\omega = k_z c_s$					
SE	$\omega \lesssim 0.5\Omega$.		streaming Max.'s		low β_i, high β_i inhibits instability
SII	$\omega \lesssim 0.5\Omega_i$	sunward $\parallel \boldsymbol{B}_0$	Spitzer–Härm		important at very low β_i
SHI	$\omega \lesssim \Omega_i$	$\sim 45°$	drifting e^-	current	wider range of θ for larger β_i
Kink, $\omega^2 = k^2 v_A^2 - k v_d \Omega_i$					
GFII		$\parallel \boldsymbol{B}_0$	drift. Max's	current	absolute instability (Re $\omega = 0$), needs elec. current
SHI			isot. drift e, i Max.'s	current	wider range of θ for larger β_i
others					
LH e^- firehose HV	$\omega \sim \Omega_i$, $kR_{Li} \gg 1$	$\parallel \boldsymbol{B}_0$	bi-Max. e, i	$T_{e\perp} < T_{e\parallel}$	$\beta \sim 1$, needs large β_e and anisotropy; ions resonant
LH mode HV	$\omega \sim \Omega_i$, $kR_{Li} \gg 1$	$\parallel \boldsymbol{B}_0$	bi-Max. e, i	$T_{i\perp} \gtrsim 3T_{i\parallel}$	ions resonant, this anisot. unlikely
RH e^- firehose HV	$\omega < \Omega_i$, $kR_{Li} \gg 1$	$\parallel \boldsymbol{B}_0$	bi-Max. e, i	$T_{e\perp} < T_{e\parallel}$	ions resonant
RH & LH firehose KS	$\omega \sim \Omega_i$	$\parallel \boldsymbol{B}_0$	general	$T_\perp < T_\parallel$	long wavelength limit anal. results
mirror GEA*c*	$\omega = 0$, $kR_{Li} < 1$	$> 45°$	bi-Max. i	$T_{i\perp} > T_{i\parallel}$	$\beta_i = 1$, may be import. if $\beta_i \gg 1$
cyclotron SHII	$\omega < \Omega_i$	$\sim 45°$	bi-Max.	$T_{i\perp} > T_{i\parallel}$	$\beta_i = 1$, only import. if $T_e \gg T_i$ (GEA*c*)
W	$\omega \approx 0$, $0.5\Omega_i$	$\perp \boldsymbol{B}_0$	2 ident. drift ion Max.	drift	
e^- beam G	$\omega \approx k v_{\text{halo}}$	$\boldsymbol{B}_0$	Max. i, drifting Max. e^-	e heat flux	more important than IA, ESIC at low T_e/T_i

References: ASF*a*, *b* = Abraham-Shrauner & Feldman (1977*a*, *b*); CS = Cornwall & Schulz (1971); DR = Drummond & Rosenbluth (1962); DT = Dobrowolny & Tessarotto (1978); F = Forslund (1970); FKK = Forslund *et al.* (1971); G = Gary (1978); GEA*a*, *b*, *c* = Gary *et al.* (1975*a*, *b*, 1976); GFI = Gary & Feldman (1977); GFII = Gary & Forslund (1975); HS = Hollweg & Smith (1976); HV = Hollweg & Völk (1970); LB = Lakhina & Buti (1976); KS = Kennel & Scarf (1968); MEA*a*, *b* = Montgomery *et al.* (1975, 1976); SI = Schwartz (1978*a*); SII = Singer (1977); SE = Schulz & Eviatar (1972); SEA = Scarf *et al.* (1967); SHI = Smith & Hollweg (1977); SHII = Soper & Harris (1965); W = Weibel (1970).

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Downloaded from rsta.royalsocietypublishing.org

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

2. Linear plasma theory

The kinetic theory of plasma microinstabilities traditionally begins by linearizing the Vlasov–Maxwell equations in terms of a small amplitude perturbation (see, for example, Akhiezer *et al.* 1975*a*; Stix 1962). (This expansion may not correctly describe the nonlinear reaction of the plasma due to the presence of finite amplitude waves, but it may be valid in regions of instability onset and initial growth. The full nonlinear problem is well beyond the capabilities of present-day theories, hence the need for some simplifying assumptions.) Figure 1

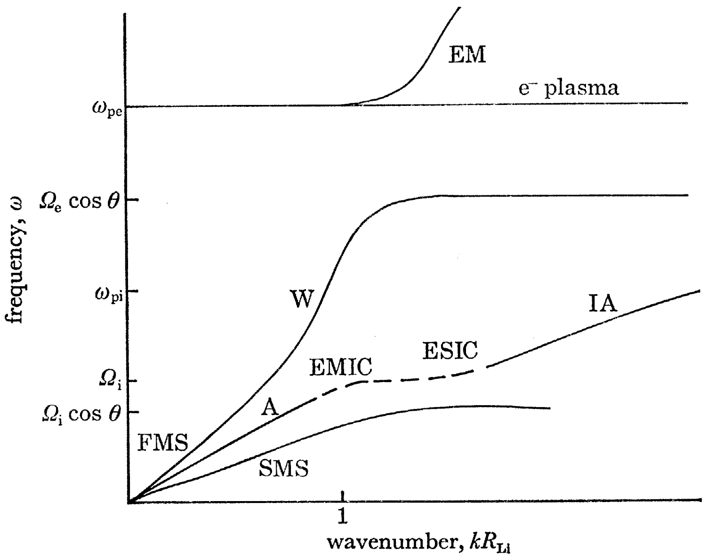

Figure 1. Obliquely propagating modes in a low-β plasma. See table 1 for key.

displays some general dispersion curves (frequency ω against wavevector k) for the normal linear Fourier modes of a low β (thermal pressure/magnetic pressure) Vlasov plasma (Boyd & Sanderson 1969). Instabilities are usually driven by resonant particles which remain in phase with a wave for many wave periods. Thus only waves with phase velocities much less than c (i.e. the lower three curves in figure 1) are relevant to the present discussion. Table 1 summarizes the vast amount of literature on the conditions for instability of these various modes. The whistler, fast mangetosonic and various cyclotron instabilities appear to be the most important for s.w. theory.

The results of linear plasma theory are usually applied to the s.w. either by searching for the mode and location at the onset of instability, where the plasma is marginally stable, or by calculating instability growth rates. The marginal stability studies (Gary *et al.* 1975*b*; Schulz & Eviatar 1972; Singer 1977; Singer & Roxburgh 1977; Smith & Hollweg 1977) not only point the direction for further work, but also have been used as a possible quasi-equilibrium state for the s.w. on the grounds that the action of instabilities must be such as to render the plasma less unstable (see, for example, Feldman *et al.* 1976). It may even be possible to construct an s.w. model on such an assumption (Rowse 1979, personal communication; Schwartz & Roxburgh 1978). The instability growth rates, on the other hand, provide information for comparison with the interplanetary wave spectrum (Scarf *et al.* 1967) and with individual instability signatures (Auer & Rosenbauer 1977). Unfortunately, all of these calculations of resonant

Downloaded from rsta.royalsocietypublishing.org

instabilities are very sensitive to the plasma model chosen (Abraham-Shrauner & Feldman 1977*a*, *b*; Schwartz 1980) so that, at present, any conclusions based on these results must be regarded as tentative.

3. Nonlinear plasma theory

Mathematically speaking, linear theory is an eigenvalue problem which determines *only* the characteristics of the normal modes of the system. Nonlinear theories are required to specify the effects of these waves on the background plasma, but nonlinear plasma physics is still in its infancy! Present-day theories are usually based either on second-order corrections to linear theory or on magnetohydrodynamic (m.h.d.) fluid equations.

The most common approach to this problem is quasilinear theory (Akhiezer *et al.* 1975*b*; Barnes 1968; Davidson 1972), which calculates the second-order changes in the background

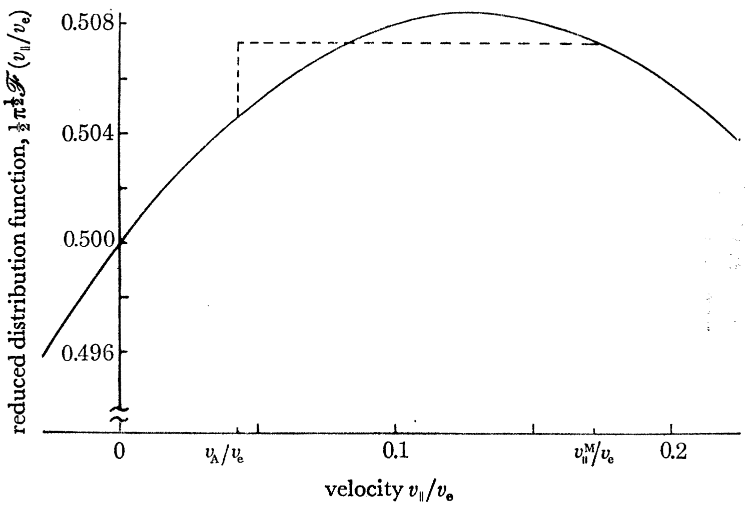

FIGURE 2. Initial (——) and plateau (– – –) forms of the normalized reduced distribution function $\mathscr{F} \propto \int d^2v_\perp f_e(\boldsymbol{v})\,(v_\perp^2 - \overline{v_\perp^2})^2$.

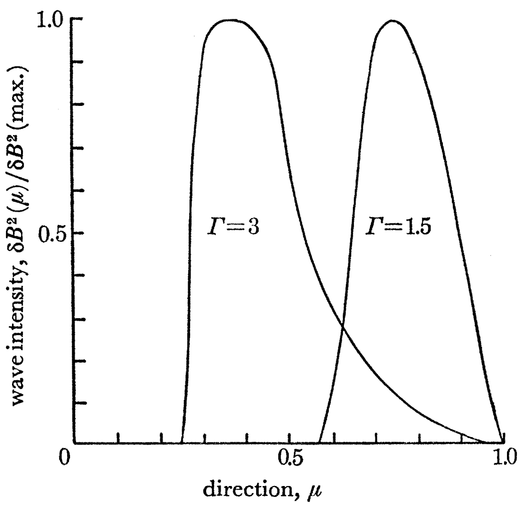

FIGURE 3. Quasilinear wave spectrum normalized to a maximum of unity against $\mu = \cos\theta$ for FMS instability. Γ measures the instability strength.

Downloaded from rsta.royalsocietypublishing.org

state due to a spectrum of waves. It ignores nonlinear interactions among the waves themselves, i.e. the waves are assumed to obey linear plasma theory. Depending on the application, this may not be too bad an approximation. Typically, quasilinear theory predicts the formation of a plateau in the distribution function at velocities which are resonant with the waves of interest. These waves then 'see' the same number of particles travelling slightly faster than themselves (these particles would tend to give up energy to the waves) as slightly slower, so that there is no net wave growth. Figures 2 and 3 show a recent example of such a calculation of plateau formation and wave spectrum for the FMS instability (Schwartz 1978*b*). The usefulness of quasilinear theory lies in its ability to estimate wave spectra, the effects of waves on the background plasma properties (heat flux, etc.), and wave-particle timescales. Quasilinear and other second-order theories, have been widely used in s.w. research (e.g. by Forslund 1970; Gary & Feldman 1977; Hollweg 1974*a*, 1978*a*; Krishan & Rankin 1972).

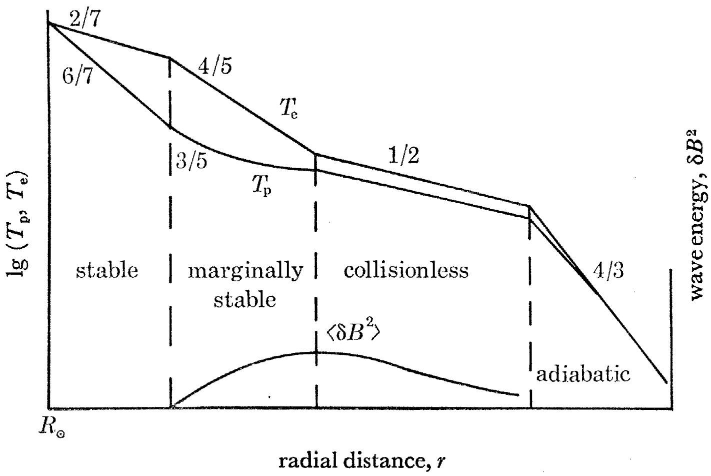

FIGURE 4. Marginally stable solar wind model.

Many other approaches to nonlinear plasma theory have been proposed, including the properties of finite amplitude m.h.d. waves (Barnes & Hollweg 1974; Hollweg 1974*b*) and kinetic waves (Abraham-Shrauner & Feldman 1977*c*; Barnes & Suffolk 1971); the steepening of large-amplitude disturbances, both in kinetic theory (Montgomery 1959) and m.h.d. (Barnes & Chao 1977; Cohen & Kulsrud 1974); nonlinear Landau damping (Hollweg 1971*b*, Lee & Völk 1973); modulational instability of large amplitude Alfvén waves (Derby 1978; Goldstein 1978; Lashmore-Davies 1976); nonlinear wave–wave interactions (Akhiezer *et al.* 1975*b*; Chin & Wentzel 1972; Cohen 1975; Cohen & Dewar 1974; Davidson 1972; Galeev & Oraevskii 1963; Harris 1969; Schwartz 1977*a*, *b*); W.K.B. wave propagation (Dewar 1970; Hollweg 1974*b*; Hollweg & Lilliequist 1978; Jacques 1977; Weinberg 1962); W.K.B. quasilinear theories (Goodrich 1978, personal communication; Hollweg 1978*a*); and strong plasma turbulence theories (Ben-Israel *et al.* 1975; Dum & Dupree 1970; Dupree 1966; Montgomery 1977; Weinstock 1969, 1970).

4. SPECULATIONS

The subject of microturbulence contains a number of potentialities for solar wind physics. Our concepts of thermal conduction and anisotropy in particular, as used in solar wind modelling, are likely to be drastically altered. Our understanding of the solar wind microstructure is,

Downloaded from rsta.royalsocietypublishing.org

obviously, completely dependent on our knowledge of the micro-processes operating in the solar wind plasma. While waiting and working for better theoretical treatments of plasma microturbulence, it is perhaps useful to speculate about the possible implications for gross solar wind properties, for example as studied via fluid models.

One such speculative model is shown in figure 4 (Schwartz & Roxburgh 1978), based on the asymptotic behaviour of the two-fluid solar wind equations with the additional assumption that the plasma remains marginally stable to the fast magnetosonic mode in those regions of the solution where linear stability theory predicts instability. Among the interesting features of this model are an elevation of the proton temperature (due to wave damping and the marginally stable part of the solution) and an accelerated flow brought about by the conversion of thermal energy, via instabilities, into bulk kinetic energy. Detailed numerical work along these lines is in progress (Rowse 1979, personal communication).

5. Conclusions

In this brief review we have tried to outline the reasons for expecting microturbulence to be a potentially important aspect of s.w. physics. We have also tried to compile a guide to the vast amount of literature on this subject. What is ultimately and desperately required is a non-local, nonlinear theory of fully developed plasma instabilities in an inhomogeneous, non-stationary, non-isotropic multi-component plasma including Coulomb collisions!

But we should not be too discouraged. The solar wind is *more* uniform, stationary, isotropic, fully ionized, infinite and collisionless than most other observable plasmas. Thus it is arguably a unique environment for testing and improving our knowledge of plasma physics.

We are grateful for numerous conversations with W. Feldman, J. Hollweg, D. Rowse, C. Singer, J. Skilling, R. Tavakol and A. Tworkowski. This work was supported, in part, by Science Research Council Grant SG/A/05631.

References (Schwartz & Roxburgh)

Abraham-Shrauner, B. & Feldman, W. C. 1977*a* *J. geophys. Res.* **82**, 1889.
Abraham-Shrauner, B. & Feldman, W. C. 1977*b* *J. Plasma Phys.* **17**, 123.
Abraham-Shrauner, B. & Feldman, W. C. 1977*c* *J. geophys. Res.* **82**, 618.
Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G. & Stepanov, K. N. 1975*a* *Plasma electrodynamics*, vol. 1: *Linear theory*. Oxford: Pergamon Press.
Akhiezer, A. I., Akhiezer, I. A., Polovin, R. V., Sitenko, A. G. & Stepanov, K. N. 1975*b* *Plasma electrodynamics*, vol. 2: *Non-linear theory and fluctuations*. Oxford: Pergamon Press.
Auer, R.-D. & Rosenbauer, H. 1977 *J. geophys. Res.* **82**, 1503.
Barnes, A. 1968 *Physics Fluids* **11**, 2644.
Barnes, A. 1979 In *Solar system plasma physics, twentieth anniversary review* (ed. C. F. Kennel, L. J. Lanzerotti & E. N. Parker). Amsterdam: North-Holland.
Barnes, A. & Chao, J. K. 1977 *J. geopyhs. Res.* **82**, 4711.
Barnes, A. & Hollweg, J. V. 1974 *J. geophys. Res.* **79**, 2302.
Barnes, A. & Suffolk, G. C. J. 1971 *J. Plasma Phys.* **5**, 315.
Ben-Israel, I., Piran, T., Eviatar, A. & Weinstock, J. 1975 *Astrophys. Space Sci.* **38**, 125.
Boyd, T. J. & Sanderson, J. J. 1969 *Plasma dynamics*. London: Thomas Nelson.
Burlaga, L. F. 1971 *Space Sci. Rev.* **12**, 600.
Chew, G. F., Goldberger, M. L. & Low, F. E. 1956 *Proc. R. Soc. Lond.* A **236**, 112.
Chin, Y. & Wentzel, D. G. 1972 *Astrophys. Space Sci.* **16**, 465.
Cohen, R. H. 1975 *J. geophys. Res.* **80**, 3678.
Cohen, R. H. & Dewar, R. L. 1974 *J. geophys. Res.* **79**, 4174.

Downloaded from rsta.royalsocietypublishing.org

Cohen, R. H. & Kulsrud, R. M. 1974 *Physics Fluids* **17**, 2215.
Cornwall, J. M. & Schulz, M. 1971 *J. geophys. Res.* **76**, 7791.
Davidson, R. C. 1972 *Methods in nonlinear plasma theory*. New York: Academic Press.
Derby Jr, N. F. 1978 *Astrophys. J.* **224**, 1013.
Dewar, R. L. 1970 *Physics Fluids* **13**, 2710.
Dobrowolny, M. & Tessarotto, M. 1978 *Astrophys. Space Sci.* **57**, 153.
Drummond, W. E. & Rosenbluth, M. N. 1962 *Physics Fluids* **5**, 1507.
Dum, C. T. & Dupree, T. H. *Physics Fluids* **13**, 2064.
Dupree, T. H. 1966 *Physics Fluids* **9**, 1773.
Feldman, W. C. 1979 In *Solar system plasma physics, twentieth anniversary review* (ed. C. F. Kennel, L. J. Lanzerotti & E. N. Parker). Amsterdam: North-Holland.
Feldman, W. C., Asbridge, J. R., Bame, S. J., Gary, S. P. & Montgomery, M. D. 1976 *J. geophys. Res.* **81**, 2377.
Forslund, D. W. 1970 *J. geophys. Res.* **75**, 17.
Forslund, D. W., Kennel, C. F. & Kindel, J. M. 1971 Plasma Physics Group Reprint R-87, Department of Physics, University of California, Los Angeles, California.
Galeev, A. A. & Oraevskii, V. N. 1963 *Soviet Phys. Dokl.* **7**, 988.
Gary, S. P. 1978 *J. Plasma Phys.* **20**, 47.
Gary, S. P. & Feldman, W. C. 1977 *J. geophys. Res.* **82**, 1087.
Gary, S. P. & Feldman, W. C. 1978 *Physics Fluids* **21**, 72.
Gary, S. P., Feldman, W. C., Forslund, D. W. & Montgomery, M. D. 1975*a* *Geophys. Res. Lett.* **2**, 79.
Gary, S. P., Feldman, W. C., Forslund, D. W. & Montgomery, M. D. 1975*b* *J. geophys. Res.* **80**, 4197.
Gary, S. P. & Forslund, D. W. 1975 *Phys. Lett.* A**54**, 347.
Gary, S. P., Montgomery, M. D., Feldman, W. C. & Forslund, D. W. 1976 *J. geophys. Res.* **81**, 1241.
Goldstein, M. L. 1978 *Astrophys. J.* **219**, 700.
Harris, E. G. 1969 In *Advances in plasma physics*, vol. 3 (ed. A. Simon & W. B. Thompson), p. 157. New York: Wiley-Interscience.
Hollweg, J. V. 1970 *J. geophys. Res.* **75**, 2403.
Hollweg, J. V. 1971*a* *J. geophys. Res.* **76**, 7491.
Hollweg, J. V. 1971*b* *Phys. Rev. Lett.* **27**, 1349.
Hollweg, J. V. 1974*a* *J. geophys. Res.* **79**, 3845.
Hollweg, J. V. 1974*b* *J. geophys. Res.* **79**, 1539.
Hollweg, J. V. 1974*c* *Publs astr. Soc. Pacif.* **86**, 561.
Hollweg, J. V. 1975 *Rev. Geophys. Space Phys.* **13**, 263.
Hollweg, J. V. 1978*a* *J. geophys. Res.* **83**, 563.
Hollweg, J. V. 1978*b* *Rev. Geophys. Space Phys.* **16**, 689.
Hollweg, J. V. & Lilliequist, C. 1978 *J. geophys. Res.* **83**, 2030.
Hollweg, J. V. & Smith, D. F. 1976 *J. Plasma Phys.* **15**, 245.
Hollweg, J. V. & Völk, H. J. 1970 *J. geophys. Res.* **75**, 5297.
Hundhausen, A. J. 1972 *Coronal expansion and solar wind*. New York: Springer-Verlag.
Jacques, S. A. 1977 *Astrophys. J.* **215**, 942.
Jockers, K. 1970 *Astron. Astrophys.* **6**, 219.
Jokipii, J. R. 1973 *A. Rev. Astr. Astrophys.* **11**, 1.
Kennel, C. F. & Scarf, F. L. 1968 *J. geophys. Res.* **73**, 6149.
Krishan, S. & Rankin, D. 1972 *Astrophys. Space Sci.* **19**, 207.
Lakhina, G. S. & Buti, B. 1976 *J. geophys. Res.* **81**, 2135.
Lashmore-Davies, C. N. 1976 *Physics Fluids* **19**, 587.
Lee, M. A. & Lerche, I. 1974 *Rev. Geophys. Space Phys.* **12**, 671.
Lee, M. A. & Völk, H. J. 1973 *Astrophys. Space Sci.* **24**, 31.
Montgomery, D. 1959 *Phys. Rev. Lett.* **2**, 36.
Montgomery, D. 1977 *Proc. Indian Acad. Sci.* A**86**, 87.
Montgomery, M. D., Gary, S. P., Feldman, W. C. & Forslund, D. W. 1976 *J. geophys. Res.* **81**, 2743.
Montgomery, M. D., Gary, S. P., Forslund, D. W. & Feldman, W. C. 1975 *Phys. Rev. Lett.* **35**, 667.
Perkins, F. 1973 *Astrophys. J.* **179**, 637.
Scarf, F. L. 1970 *Space Sci. Rev.* **11**, 234.
Scarf, F. L., Wolfe, J. H. & Silva, R. W. 1967 *J. geophys. Res.* **72**, 993.
Schulz, M. & Eviatar, A. 1972 *Cosmic Electrodyn.* **2**, 402.
Schwartz, S. J. 1977*a* Ph.D. thesis, University of Cambridge.
Schwartz, S. J. 1977*b* *Mon. Not. R. astr. Soc.* **178**, 399.
Schwartz, S. J. 1978*a* *J. geophys. Res.* **83**, 3745.
Schwartz, S. J. 1978*b* Quasilinear saturation of fast magnetosonic instabilities in the solar wind. Unpublished manuscript.
Schwartz, S. J. 1979 In *Lecture notes in physics*. Berlin, Heidelberg and New York: Springer-Verlag. (In the press).
Schwartz, S. J. 1980 *Rev. Geophys. Space Phys.* (In the press.)

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Downloaded from rsta.royalsocietypublishing.org

Schwartz, S. J. & Roxburgh, I. W. 1978 *Proc. 2nd European Solar Phys. Meeting on Highlights of Solar Physics*, 8–10 March 1978, Toulouse, France (Éditions du Centre National de la Recherche Scientifique), p. 317.
Singer, C. 1977 *J. geophys. Res.* **82**, 2686.
Singer, C. & Roxburgh, I. W. 1977 *J. geophys. Res.* **82**, 2677.
Smith, D. F. & Hollweg, J. V. 1977 *J. Plasma Phys.* **17**, 105.
Soper, G. K. & Harris, E. G. 1965 *Physics Fluids* **8**, 984.
Spitzer Jr, L. & Härm, R. 1953 *Phys. Rev.* **89**, 977.
Stix, T. H. 1962 *The theory of plasma waves*. New York: McGraw-Hill.
Völk, H. J. 1975 *Space Sci. Rev.* **17**, 255.
Weibel, E. S. 1970 *Physics Fluids* **13**, 3003.
Weinberg, S. 1962 *Phys. Rev.* **126**, 1899.
Weinstock, J. 1969 *Physics Fluids* **12**, 1045.
Weinstock, J. 1970 *Physics Fluids* **13**, 2308.

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF

MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES
THE ROYAL SOCIETY A
PHILOSOPHICAL TRANSACTIONS OF